MATHEMATICS

Time: Two Hours   **Max.Marks: 60**
(8.30 AM – 10.30 AM)

NOTE:-

1. *Attempt all questions.*
2. *Rough work must be enclosed with answer book.*
3. *While answering, refer to a question by its serial number as well as section heading. (eg.Q2/Sec.A)*
4. *There is no negative marking.*
5. *Answer each of Sections A, B, C at one place.*
6. *Elegant solutions will be rewarded.*
7. *Use of calculators, slide rule, graph paper and logarithmic, trigonometric and statistical tables is not permitted.*

Note:- All answers to questions in **Section-A, Section-B** and **Section-C** must be supported by mathematical arguments. In each of these sections order of the questions must be maintained.

SECTION-A

This section has Six Questions. Each question is provided with five alternative answers. Only one of them is the correct answer. Indicate the correct answer by A, B, C, D, E.

                                                                                                    (6x2=12 MARKS)

1. Let A= {1, 2, 3, …., 2008}, B = {1, 2, 3, ….., 1004}.   [   ]
   A) There can be infinitely many functions from A to B
   B) There can not be an onto function from A to B
   C) There can be at least one one-one function from A to B
   D) There can be infinitely many onto functions from A to B
   E) None of these

2. The number of equiangular octagons fixing 6 consecutive sides is   [   ]
   A) Infinitely many
   B) exactly 8
   C) at most 8
   D) 0
   E) None of these

3. All the numbers between 1947 and 2008 are written, including 1947 and 2008. From the list, all the multiples of 3 and 5 are struck off. The sum of the remaining numbers is   [   ]
   A) 41517   B) 73137   C) 73138   D) 65247   E) 65248

4. A square ABCD is inscribed in a quarter circle where B is on the circumference of the circle and D is the center of the circle. The length of diagonal AC of the square, if the circle's radius is 5, is   [   ]
   A) $5\pi/4$ B) $5\pi/2$ C) 5   D) $5\sqrt{2}$
   E)The length cannot be determined.

5. 50 x 50 x 50 x … (where there are a hundred 50s) is how many times 100 x100 x 100 x … (where there are fifty 100s)? [   ]
   A) 25 x 25 x 25 x …(where there are fifty 25s)
   B) 4 x 4 x 4 x … (where there are fifty 4s)
   C) 2 x 2 x 2 x … (where there are fifty 2s)
   D) 1 time
   E) None of these

6. a, b are positive integers. A is the set of all divisors of 'a' except for 'a'. B is the set of all divisors of 'b'. If A = B then which of the following is a <u>wrong</u> statement? [   ]
   A) $a \geq b$
   B) a is a multiple of b
   C) b is a not a multiple of a
   D) a/b is a prime number
   E) none of these

SECTION-B

This section has Six Questions. In each question a blank is left. Fill in the blank.
(6x2=12 MARKS)

1. The domain of the real function $f(x) = \sum_{k=1}^{2008} \frac{1}{\sqrt{|x-k|}}$ is _____

2. The number of points P strictly lying inside an equilateral triangle ABC such that the sum of the perpendicular distances from P to the three sides of the triangle is minimum is_____

3. Positive integers a, b are such that both are relatively prime and less than or equal to 2008, $a^2 + b^2$ is a perfect square and that b has the same digits as a in the reverse order. The number of such ordered pairs (a, b) is _____

4. Let ABCD be a square. E, F, G, H be the mid points of its sides $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ respectively. Let P, Q, R, S be the points of intersection of the line segments $\overline{AF}$, $\overline{BG}$, $\overline{CH}$, $\overline{DE}$ inside the square. The ratio of the areas $\triangle PQRS : \triangle ABCD$ is _____

5. The number of elements in the set {(a, b, c) : a, b, c are three consecutive integers in some order, a + b + c = abc} is _____

6. The sum of all positive integers for which the quotient and remainder are equal if the number is divided by 2008 is _____

SECTION-C

(6x2=12 MARKS)

1. Is y a real function of x in the equation |y+2008| = $x^2 + 2$?

2. The people living on Street 'S' of Y-City all decide to buy new house numbers so they line up at the only Hardware store in order of their addresses: 1, 2, 3, ... If the store has 100 of each digit, what is the first address that won't be able to buy the digits for its house number?

3. Let ABCD be a quadrilateral such that AB is perpendicular to BC, AD is perpendicular to BD and AB=BC, BD = a, AD = c, CD = x. Find x in terms of a and c.

4. For what pair wise different positive integers is the value of $\dfrac{a}{a+1} + \dfrac{b}{b+1} + \dfrac{c}{c+1} + \dfrac{d}{d+1}$ an integer?

5. Side *AB* of rectangle *ABCD* is 2 units long and side *AD* is 3 units. E is a point on the line AC such that C is the mid point of the line segment AE. What is the length of line segment *BE*?

6. How many number of integers are there between 2008 and 2,00,82,008 including 2008 and 2,00,82,008 such that the sum of the digits in the square is 42?

SECTION-D

(6x4=24 MARKS)

1. Let S = {1, 2, 3, …., 2008}. Find the number of elements in the set {(A, B) : A∪B = S}

2. A square is drawn in side a triangle with sides 3, 4 and 5 such that one corner of the square touches the side 3 of the triangle, another corner touches the side 4 of the triangle, and the base of the square rests on the longest side of the triangle. What is the side of the square?

3. State and prove the test of divisibility of a positive integer 'a' by 11.

4. A square cake 6" x 6" and 3" tall was cut into four pieces of equal volume as shown in the figure. Determine how far in from the side of the cake the cuts should be made? (i.e., x = ?)

Cake
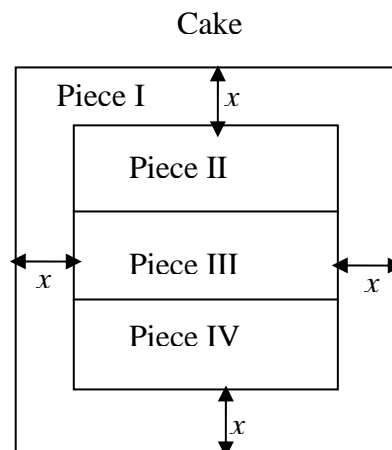
5. Solve the following simultaneous equations for a and b:
$$a\sqrt{a} + b\sqrt{b} = 183, \qquad\qquad a\sqrt{b} + b\sqrt{a} = 182$$

6. f and g are two real variable real valued functions defined by
$$f(x) = \begin{cases} x+2, & if \ x \ge 0 \\ 2x-3, & if \ x < 0 \end{cases}, \quad g(x) = \begin{cases} 3x+7, & if \ x \ge 2 \\ x-1, & if -2 < x < 2 \\ x+1, & if \ x \le -2 \end{cases}. \text{ Find gof.}$$