**Time: Two Hours** **MATHEMATICS** **Max.Marks: 50**
**(8.30 AM – 10.30 AM)**

**NOTE:-**

1. *Attempt all questions. Rough work must be enclosed with answer book.*
2. *While answering, refer to a question by its serial number as well as section heading. (eg.Q2/Sec.A)*
3. *There is no negative marking.*
4. *Answer each of Sections A, B, C at one place. Elegant solutions will be rewarded.*
5. *Use of calculators, slide rule, graph paper and logarithmic, trigonometric and statistical tables is not permitted.*

**Note:-** All answers to questions in **Section-A, Section-B** and **Section-C** must be supported by mathematical arguments. In each of these sections order of the questions must be maintained.

## SECTION-A

**This section has Five Questions. Each question is provided with five alternative answers. One or more than one of them are correct answers. Indicate the correct answers by A, B, C, D, E.** (5x2=10 MARKS)

1. Let $l_1$, $l_2$ be any two parallel lines and B, C be any two points on $l_1$ and $A_1$, $A_2$, …., $A_{2010}$ be points on $l_2$. If $\Delta_i$ denotes the area of the triangle $A_iBC$ and if $\sum_{i=1}^{2010} \Delta_i = 2010$, Then the area of $\Delta A_{2010}BC$ is

   A) 1 B) ½ C) 2 D) 2010 E) 1005

2. Let $\{a_n\}$ be a sequence of integers such that $a_1 = 1$, $a_{m+n} = a_m + a_n + mn$ for all positive integers m and n. Then $a_{12}$ is
   A) 6 B) 70 C) 78 D) 76 E) 72

3. In a triangle ABC, a, b, c denote the lengths of the sides BC, CA, AB. If D is the midpoint of the side BC and AD is perpendicular to AC, then
   A) $3b^2 = a^2 - c^2$ B) $3a^2 = b^2 - 3c^2$ C) $b^2 = a^2 - c^2$ D) $a^2 + b^2 = 5c^2$ E) none of these

4. If k is an integer then which of the following is true?
   A) An integer of the form 4k+1 can always be put in the form 2k-1
   B) An integer of the form 4k+3 can always be put in the form 2k+1
   C) An integer of the form 2k-1 can always be put in the form 4k+1
   D) An integer of the form 2k-1 can always be put in the form 4k+3
   E) An integer of the form 2k+1 can always be put in the form 4k+3

5. The number of elements in $\{(a, b, c) / a=b, (a-c)^2 = 0, a+b+c=0, a, b, c \text{ are real numbers}\}$ is
   A) 0 B) 1 C) 6 D) 3 E) infinitely many

## SECTION-B

**This section has Five Questions. In each question a blank is left. Fill in the blank.** (5x2=10 MARKS)

1. The no. of solutions of the equation $xy(x+y)=2010$, where $x$ and $y$ denote positive prime numbers, is_____
2. The number of elements in the set $\{n \in N / n^3-8n^2+20n-13$ is a prime number$\}$ is _____
3. The solution set of the equation $\sqrt{x^2 - 4x + 4} + (x-2) = 0$ is _____
4. Given any two diameters of a circle the convex quadrilateral formed by joining the extremities of the diameters is always a rectangle. True/False _____
5. If $P = 3^{2010} + 3^{-2010}$, $Q = 3^{2010} - 3^{-2010}$ then $P^2 - Q^2$ = _____

## SECTION-C

(5x2=10 MARKS)

1. Solve the equation $\log_{2010}(2009x) = \log_{2009}(2010x)$.
2. In a quadrilateral ABCD, AB = 3, BC = 4, CD = 5, $\angle$ ABC = $\angle$ BCD = $120^0$. Find the area of the quadrilateral.
3. I was trying to solve $\frac{4}{x-2} > 5$. While writing the question I mistakenly wrote a digit other than 5 and solved the inequality and got $2<x<4$. What digit did I write possibly?
4. In a right angled triangle what is the relation between the square of the altitude on to the hypotenuse and the product of the segments of the hypotenuse?
5. Is it possible to find two functions f and g such that the domain of f is <u>not</u> finite, the domain of g is finite, gof is defined? Justify your answer.

## SECTION-D

(5x4=20 MARKS)

1. If the last digits (unit places) of the products 1.2., 2.3, 3.4,.., n(n+1) are added, the result is 2010. How many products are used?
2. Show that four divides any perfect square or leaves a remainder 1. Also show that nine divides cube of any integer or leaves 1 or 8 as remainder.
3. Let AB be a line segment of length 26. Let C and D be located on the line segment AB such that AC = 1 and AD = 8. Let E and F be the points on one of the semi circles with diameter AB for which EC and FD are perpendicular to AB. Find the length of the line segment EF.
4. In each of the following cases give an example of a system of two linear equations in two variables x and y.
   i) A system having exactly one solution ii) A system having no solution
   iii) A system having infinitely many solutions
5. Using Mathematical Induction Prove that $3^{2n} + 7$ is divisible by 8, $\forall$ $n \in N$.