MATHEMATICS

Time : Two Hours **Max. Marks: 60**
(8.30 AM – 10.30 AM)

NOTE:-

1. Attempt all questions.
2. Answer to each question under Section D should begin on a fresh page and rough work must be enclosed with answer book.
3. While answering , refer to a question by its serial number as well as section heading.(eg. Q2/Sec.A)
4. There is no negative marking
5. Answer each of Sections A, B, C at one place.
6. Elegant solutions will be rewarded.
7. Use of calculators, slide rule, graph paper and logarithmic, trigonometric and statistical tables is not permitted.

Note:- All answers to questions in Section – A, Section – B, Section – C must be supported by mathematical arguments. In each of these sections order of the questions must be maintained.

SECTION - A

This section has Six Questions. Each question in provided with five alternative answers, Only one of them is the correct answer. Indicate the correct answer by A, B, C, D, E.

(6×2 = 12 MARKS)

1. C is a circle and P is a point exterior to it. Several lines are drawn through P such that each line has nonempty intersection with C. If 2005 points of intersection are formed, then among the lines [ ]

A) there need not be even one tangent
B) there may be two tangents
C) there may be three tangents
D) there has to be precisely one tangents
E) none of these

2. For i = 1, 2, …., 2005, a_i is a positive integer a_n, a_{n+1}, a_{n+2} is a G.P. if n odd. a_n, a_{n+1}, a_{n+2} is an A.P. if n is even. [ ]
A) there is no such sequence
B) $a_i = 1$ for all $i \in \{1, 2, …., 2005\}$
C) there are only a finite number of such sequences
D) a_n/a_1 is the square of a rational number, if n is odd
E) none of these

3. P is an interior point of a circle C whose centre is O and $O \neq P$. C_1 is the circle with $\overline{OP}$ as diameter. Q is a point on the circumference of C_1 such that $Q \notin \{O, P\}$. $\overleftrightarrow{PQ}$ intersects C in A and B. Then AQ/QB is
[ ]
A) independent of P and Q
B) independent neither of P nor of Q
C) independent of P but not of Q
D) independent of Q but not of P
E) dependent on the radius of C

4. n is a positive integer. The number of its positive integral divisors is 2005. Then [ ]

A) it cannot be the 4-th power of a positive integer
B) it cannot be the 3-rd power of a positive integer
C) it cannot be the 12-th power of a positive integer
D) it cannot be the 6-th power a positive integer
E) none of these

5. D, G are points on the side $\overline{AB}$ of Δ ABC. E and F are points on the sides $\overline{AC}$ and $\overline{BC}$ respectively such that $\overline{DE} // \overline{BC}$, $\overline{EF} // \overline{AB}$ and $\overline{FG} // \overline{CA}$. Then D, E, F, G are the consecutive vertices of a quadrilateral [ ]

A) always
B) only if $\frac{AD}{AB} > \frac{1}{2}$
C) only if $\frac{AD}{AB} = \frac{1}{2}$
D) only if $\frac{AD}{AB} < \frac{1}{2}$
E) none of these

6. Let S be the set of points common to the lines ax + by = p, cx + dy = q. Let S' be the set of points common to the lines dx + by = p' and cx + ay = q'. Then : [ ]
A) if S is empty then S' is empty
B) if S is infinite so is S'
C) if S consists of only one element, then S' is empty
D) if S consists of only one element, then S' is infinite
E) if S consists of only one element, so does S'.

SECTION - B

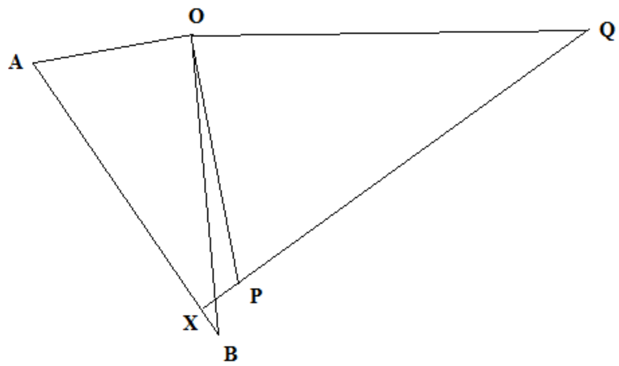
This section has Six Questions. In each question a blank is left, Fill in the blank.
(6×2 = 12 MARKS)

1. The number of positive integers n such that 2005 is a divisor of $n^2 + n + 1$ is __________
2. $I_1, I_2, \ldots, I_{2005}$ are arcs of circles. I_k is part of a circle with radius r_k and subtends an angle θ_k radians at the centre. C is the circle with radius $r_1 + r_2 + \ldots + r_{2005}$. The arc of C whose length is the sum of the lengths of $I_1, I_2, \ldots, I_{2005}$ subtends at the centre of C an angle of __________________ radians.
3. The greatest positive integer k such that $x^k - 1$ is a divisor both of $x^{2005} - 1$ and $x^{1203} - 1$ is ______________
4. The distance between the parallel sides $\overline{AB}$ and $\overline{CD}$ of a trapezium is 12 units. AB = 24 units, CD = 15 units. E is the midpoint of $\overline{AB}$. O is the point of intersection of $\overline{DE}$ with $\overline{AC}$. The area of the quadrilateral EBCO is __________________
5. The set of all natural numbers n such that n $\geq$ 2 and 2005 is the sum of n consecutive natural numbers is _______________
6. Consider the smallest multiple of 2005 such that its digits are the lengths of the sides of a pentagon. The number of unit sides in the pentagon is ___________________

SECTION - C

(6×2 = 12 MARKS)

1. C is in the interior of $\angle$ AOB. Locate a point P on $\vec{OA}$ and Q on $\vec{OB}$ such that C is the midpoint of $\overline{PQ}$.
2. P(x) is a polynomial with integral coefficients. P(x) = 4010 for 5 different integral values of x. Prove that there is no integer x such that P(x) = 2005.
3. Determine the radius of the circle inscribed in a rhombus whose diagonals measure 10 units and 24 units.
4. There are 15 sets of lines in a plane, one consisting of 77 parallel lines, another 5 parallel lines, another 4 parallel lines and another 3 parallel lines. The remaining sets consist of one line each. If no two lines are coincident, no three of them are concurrent and lines belonging to different sets intersect, determine the points of intersection.
5. In two triangles Δ ABC and Δ DEF, AB = DE, AC = DF, $\angle$ ACB, $\angle$ DEF are of equal measure. Is it necessary that $\angle$ ABC and $\angle$ DEF are of equal measure? Discuss.
6. In the adjoining figure Δ OAB $\equiv$ Δ OPQ in the indicated order of correspondence. $\vec{QP}$ meets $\overline{AB}$ in X. Prove that $\overleftrightarrow{OX}$ bisects $\angle$ AXP.



SECTION - D

(6×4 = 24 MARKS)

1. Factorize $2(a-b)^2 + (b-c)(c-a)$.
2. In a Δ ABC, $\angle$ ABC = 7^0. Explain how you locate all points P in the plane of the triangle such that $\angle$ APC = 112^0.
3. $x \neq 0$. If $u_1 = 0$ and if $u_{n+1} = (1-x)u_n + nx$ for all natural numbers n, then prove by induction that $u_n = \frac{1}{x}\{nx - 1 + (1-x)^n\}$ for all natural numbers n.
4. A, B, C are three non collinear points. Explain how you will draw a circle with centre at C such that parallel tangents can be drawn from A and B.
5. Given positive integers $a_0, a_1, a_2, \ldots, a_{2005}$. It is known that $a_1 > a_0$, $a_2 = 3a_1 - 2a_0$, $a_3 = 3a_2 - 2a_1, \ldots, a_{2005} = 3a_{2004} - 2a_{2003}$. Prove that $a_{2005} > 2^{2004}$
6. $\overline{AB}$ is a diameter of a circle, π_1, π_2 the two half – planes determined by $\overleftrightarrow{AB}$. $P_1, P_2, \ldots, P_{2004}$ are points on $\overline{AB}$ such that $AP_1 = P_1P_2 = P_2P_3 = \ldots = P_{2003}P_{2004} = P_{2004}B$. For every r $\in$ {1, 2, 3, …., 2004} draw a hook, which is the figure formed by the semicircle in π_1 on $\overline{AP_r}$ as diameter and the semicircle in π_2 on $\overline{P_rB}$ as diameter. Prove that these hooks divide the circle into 2005 regions of equal area.